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Finite element analysis of curved beams

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Abstract

Curved beam structures play a vital role in many engineering applications, such as civil, mechanical, and aerospace engineering. Finite Element Analysis of curved beams poses unique challenges to obtain exact stiffness and mass matrices due to the combined effects of axial, shear and bending forces, along with geometric nonlinearity and material behavior. In this study, finite curved beam element and finite straight beam element were used to simulate a quarter-circular cantilever ring beam. This paper investigates the effect of the radius of curvature on axial, shear and rotational displacements obtained by the two approaches. A comparison between using the finite curved beam element and the finite straight beam element shows that the two methods provide very similar results when the curve radii are not big.

Keywords: FEA, straight beam elements, curved beam elements, radius of curvature.

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تحليل العناصر المحدودة للجوائز المنحنية

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الملخص

تلعب المنشآت ذات الجوائز المنحنية دورًا حيويًا في العديد من التطبيقات الهندسية، مثل الهندسة المدنية والميكانيكية و الطيران. وتُـواجه طريقة التحليل بإستخدام العناصر المحدودة للمنشآت المحتوية على جوائز منحنية تحديًا فريدًا وذلك للحصول على مصفوفات الصلابة والكتلة الصّحيحة والدقيقة بسبب التأثيرات المشتركة للقوى المحورية وقوى القص وقوى الانحناء، إلى جانب سلوك المواد اللاخطي. في هذه الدراسة، تم استخدام طريقة عنصر جائز منحني محدود وطريقة عنصر جائز مستقيم محدود لمحاكاة عارضة حلقية ربع دائرية مثبتة من جهة واحدة. في هذه الورقة البحثية سيتم التحقق من تأثير نصف قطر الانحناء على الإزاحات المحورية والقصية والدورانية بإستخدام الطريقتين. تُظهر نتائج المقارنة بين استخدام طريقة عنصر جائز منحني محدود وطريقة عنصر جائز مستقيم محدود لتحليل جائز حلقي ربع دائري، أن الطريقتين تُعطيان نتائج قريبة جدًا من بعضهما البعض عندما لا يكون نصف قطر الانحناء كبير.

الكلمات المفتاحية: تحليل العناصر المحدودة (FEA)، عناصر الجائز المستقيم، عناصر الجائز المنحني، نصف قطر الانحناء.

1- Introduction

Finite element method (FEM) is one of the most powerful numerical techniques that can be applied to obtain approximate numerical solutions to a wide range of problems in various fields of engineering. Indeed, formulation errors are one of the main sources of errors in FEM, which results from the use of FEM that do not precisely describe the element's behaviour [1].

When finite element analysis is used for a structure that involves a curved beam, the simplest way of dealing with the curved beam, is usually to divide it into a series of finite straight beam elements to approximate the true curved shape. With this procedure, stiffness matrix for static analysis and mass matrix for dynamic analysis of finite straight beam element are utilized and the result is approximated [2-5]. However, some researchers to some extent disagree to use finite straight beam segments formulation to numerically simulate the curved beam, because in its formulation both the contribution of radial stresses and the Jacobian in the volume integral are neglected, which is a problem when a large size of curved structural beam is tackled [6-8].

The other solution to model curved beams is to use finite curved elements, but this is often coupled with complex formulations for the element property matrices and heavy numerical computations [9-11]. As a result, using finite curved element method to simulate curved beams is not preferred and so frequently causes frustration to finite element analysts.

The main purpose of the research is to investigate the effect of the radius of curvature on axial, shear and rotational displacements obtained by using the finite straight beam element method (SE-method) and finite curved beam element method (CE-method). The investigation is carried out on a quarter-circular cantilever ring beam and the displacement results of the two methods are compared.

2- FEA procedures

The finite element method can be described systematically through a step-by-step process. The following sequence of steps provides a concise summary of the FE analysis procedure [1]:

1. Domain discretization: The first step in the FEM involves dividing the solution domain into finite elements connected to each other at points called nodes. The number of elements should not be large to reduce computational time.
2. Selection interpolation functions: Choose the appropriate interpolation function to represent the variation of the field

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variable within each element based on the values at the nodes.

3. Find the element properties: Calculate local stiffness and mass matrices that express the properties of each element.
4. Assembly of element properties: Combine local mass/stiffness matrix of each element to obtain the global matrix for the entire structure.
5. Apply boundary conditions: Impose the boundary conditions to the model.
6. Solution: Solve the system of equations to determine the unknown nodal quantities such as displacement.
7. Post Processing of local values.

3- Finite straight beam elements

Consider a 2D plane straight beam element, as shown in figure 1. This beam element can be subjected to axial loading (F_{X1}, F_{X2}), shearing forces (F_{Y1}, F_{Y2}) and bending moments (M_1, M_2).

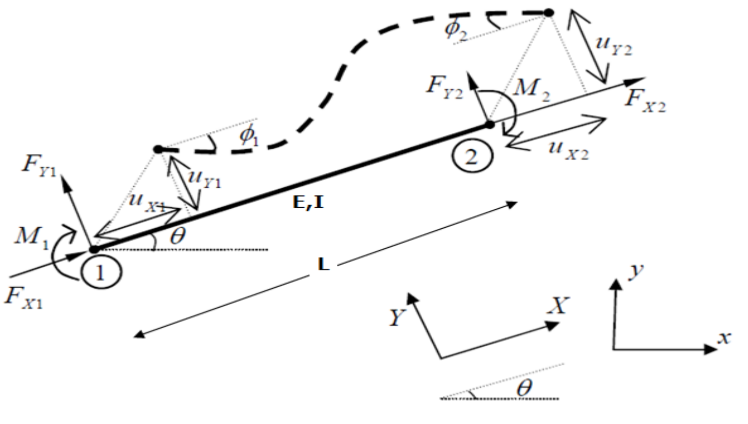

Figure 1: Finite straight beam element.

The stiffness matrix in local coordinates (**K**$_e$) is expressed by the following equation:

$$\mathbf{K}_e = \int_{V_e} \mathbf{B}^T \mathbf{D}_{el} \mathbf{B}\, dV^e = \int_{-1}^{+1} \int_{-1}^{+1} \mathbf{B}^T \mathbf{D}_{el} \mathbf{B}\, \det\mathbf{J}\, t_e\, d\xi d\eta \qquad (1)$$

After the integration, equation (1) becomes:

$$\mathbf{K_e} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^2} & \frac{-6EI}{L^2} & 0 & \frac{-12EA}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \qquad (2)$$

Where (L) is length of the beam element, (I) is moment of inertia of the cross-sectional area, and (E) is Elastic Modulus.

The assembled stiffness matrix for the structure is expressed as:

$$\mathbf{Kg} = [T]^T [\mathbf{K_e}] [T] \qquad (3)$$

[T]is the transformation matrix that relates element nodal variables to the global nodal variables.

4- Finite curved beam element

The geometry of a curved beam element is shown in figure 2. The formulations of stiffness and mass matrices derived by [5], were employed to numerically simulate the behavior of curved beams. The curved element has radial displacement u_x, circumferential displacement u_θ, and a rational angle ψ_y .

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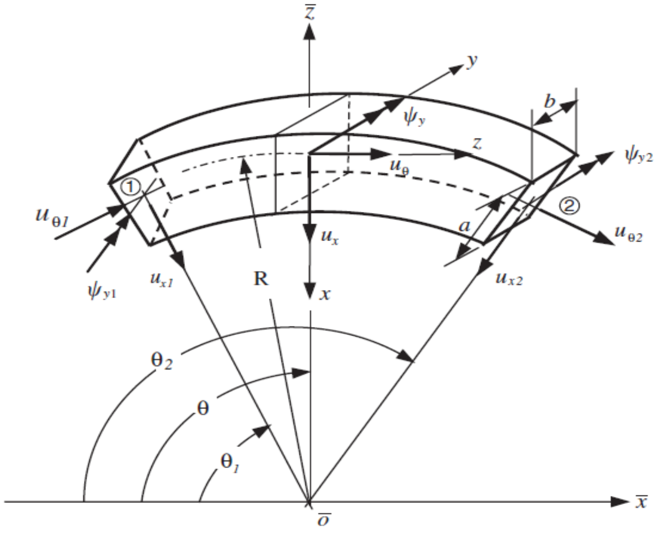

Figure 2: Curved beam element [4].

The local stiffness matrix KL of the curved element, shown in Figure 3, is obtained as follows:

$$Ke= [W]\,[B]^{-1} \tag{4}$$

The matrix D and matric B are given by:

$$W=\frac{EI_y}{R^2}\begin{bmatrix} 0 & 0 & 0 & -\frac{2}{R}\sin\theta_1 & 0 & -\frac{2}{R}\cos\theta_1 \\ 0 & 0 & 0 & \frac{2}{R}\cos\theta_1 & 0 & -\frac{2}{R}\sin\theta_1 \\ -1 & 0 & 0 & -2\cos\theta_1 & 0 & -2\sin\theta_1 \\ 0 & 0 & 0 & \frac{2}{R}\sin\theta_2 & 0 & \frac{2}{R}\cos\theta_2 \\ 0 & 0 & 0 & -\frac{2}{R}\cos\theta_2 & 0 & \frac{2}{R}\sin\theta_2 \\ 1 & 0 & 0 & 2\cos\theta_2 & 0 & -2\sin\theta_2 \end{bmatrix} \tag{5}$$

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$$B = \begin{bmatrix} 1 & \cos\theta_1 & \sin\theta_1 & \theta_1 \sin\theta_1 & 0 & \theta_1 \cos\theta_1 \\ 0 & \sin\theta_1 & -\cos\theta_1 & \sin\theta_1 - \theta_1 \cos\theta_1 & 1 & \cos\theta_1 + \theta_1 \sin\theta_1 \\ \frac{C}{R}\theta_1 & 0 & 0 & \frac{2}{R}\sin\theta_1 & \frac{1}{R} & \frac{2}{R}\cos\theta_1 \\ 1 & \cos\theta_2 & \sin\theta_2 & \theta_2 \sin\theta_2 & 0 & \theta_2 \cos\theta_2 \\ 0 & \sin\theta_2 & -\cos\theta_2 & \sin\theta_2 - \theta_2 \cos\theta_2 & 1 & \cos\theta_2 + \theta_2 \sin\theta_2 \\ \frac{C}{R}\theta_2 & 0 & 0 & \theta_2 \sin\theta_2 & \frac{1}{R} & \frac{2}{R}\cos\theta_2 \end{bmatrix} \quad (6)$$

Where: R is the average radius of the beam curvature and I_y is the moment of inertia of the area A about the y-axis. The equation 3 is used to assembly the global stiffness matrix Kg. Then the nodal displacements are computed and ordered as shear, axial and moment respectively.

5- Numerical examples

The quarter-circular cantilever ring beam shown in figure 4 was used in this finite element study to investigate to effect of the radius of curvature. Twenty different values of radius of curvature R were studied, started from 0.5m to 10m with 0.5m increment. A MATLAB code was written for the purpose of this numerical investigation. Figure 3 shows the flow chart of the MATLAB code.

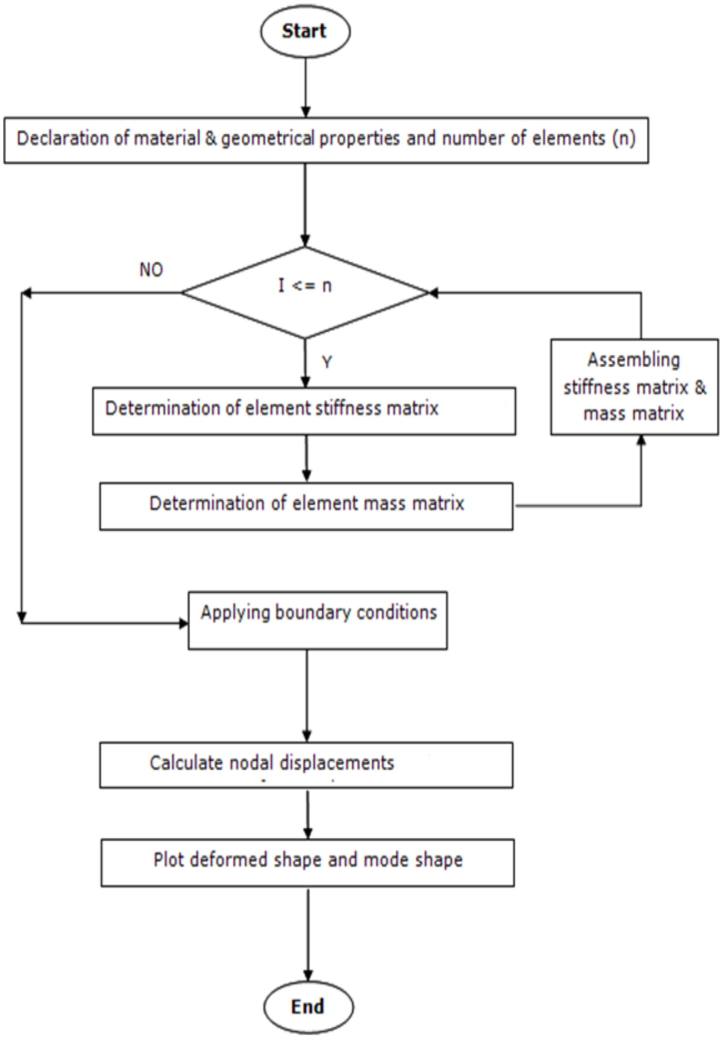

Figure (3): Flow-chart of the MATLAB Code

The quarter cantilever beam is subjected to a vertical load at its free end (*P)*, as you can see in figure 4. The finite element results for axial displacement (U), shear displacement (W) and rotational displacement (ϴ) at the ring tip are computed by finite curved beam element formulations (CE-method) and finite straight beam element

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formulations (SE-method) over the range of radius of curvature. The following data are used:

- Radius of curvature **R =** 0.5m to 10m with 0.5m increment.
- Elastic Modulus **E**= 200 GPa.
- Vertical load ***P***= 1 N.
- Mass density **ρ**=7850 Kg/m^3.

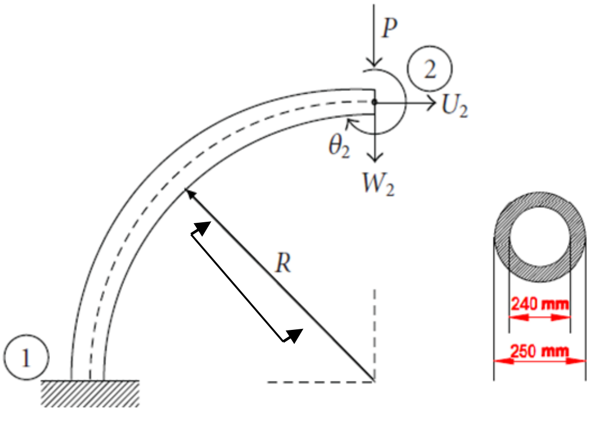

Figure (4): A quarter-circular cantilever ring beam

It is worthwhile to mention here that:

I) The number of elements was increased until there was no observable change in the value of displacements even when the number of elements increased.

II) The hypothesis of the research is that, CE-method gives the exact solution, while SE-method approximates the solution.

III) The axial, shear and rotational displacements of the quarter-circular cantilever beam were calculated at its tip.

Figures 5, 6, and 7 shows the influence of radius of curvature of the quarter-circular cantilever beam on axial, shear and rotational displacements, respectively.

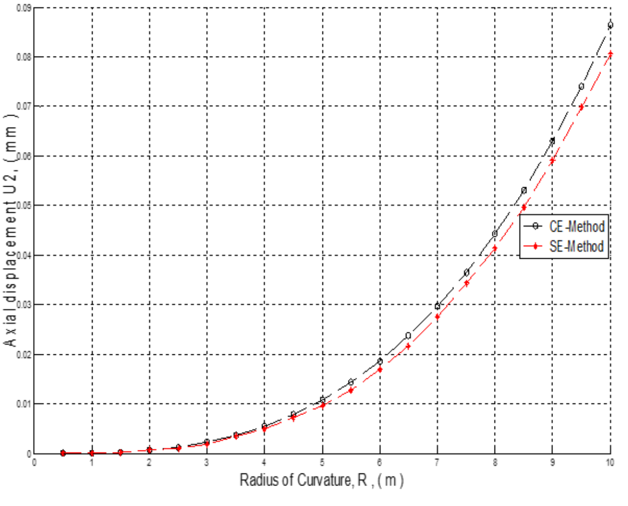

Figure (5): Effect of the radius of curvature on the axial displacement of the quarter-circular cantilever beam.

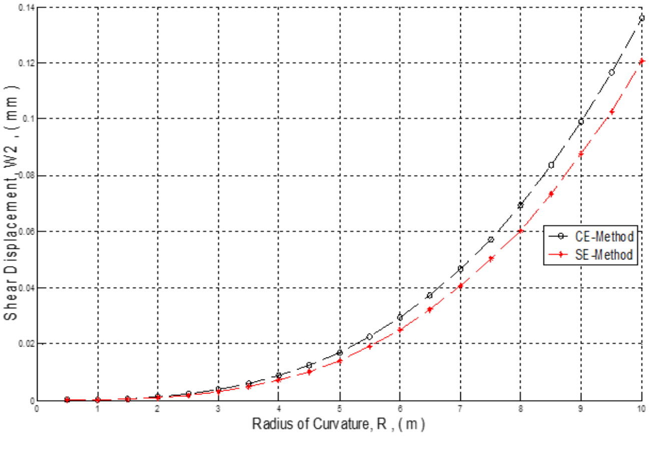

Figure (6): Effect of the radius of curvature on the shear displacement of the quarter-circular cantilever beam.

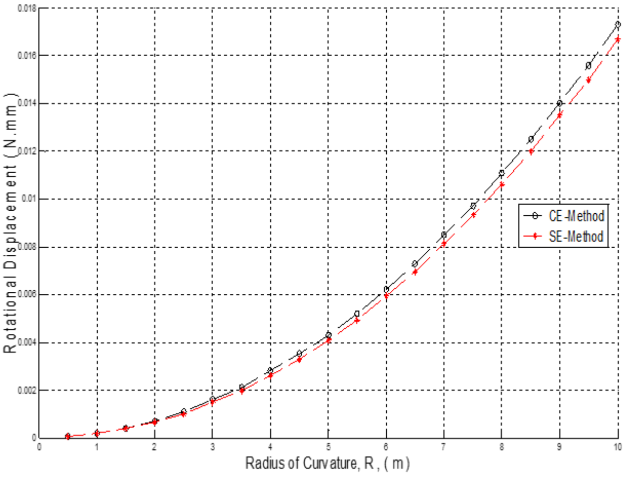

Figure (7): Effect of the radius of curvature on the rotational displacement of the quarter-circular cantilever beam.

As shown in figures (5) and (7), the axial and rotational displacements of the quarter-circular cantilever beam from SE-method are close to those obtained by CE-method. However, it should be noted that there is a slight increase in differences between the results of the two methods when the radius of curvature increases.

The relationship between the radius of curvature of the quarter circular cantilever beam and its tip shear displacement is depicted in figure (6). It was found that when the radius of curvature ranged from 0.5m to 5m, the results of shear displacement that obtained by the two finite element methods were close. However, the closeness between the results of shear displacement from CE-method and SE-method is disproportional to the increase in the radius of curvature of the quarter circular beam.

6- CONCLUSION

The effect of the radius of curvature on the axial, shear and rotational displacements of a quarter circular cantilever beam was studied by simulating the curvature as a series of straight beam

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elements approach (SE-method) and a curved finite beam elements approach (CE-method). Conclusions drawn from this investigation are as follows**:**

- The results of displacements of the quarter circular cantilever beam obtained by the finite straight beam element formulations (SE-method) finite curved beam element formulations (CE-method) are close to those obtained from the finite curved beam element formulations (CE-method) when the radius of curvature is not big, less than 5 meters.
- It is evident that the closeness of displacements results between the two methods decreases when the radius of curvature is more than about 5 meters. Consequently, finite curved beam element formulations should be used for analysing structures that involves curved beams with a large radius of curvature.

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